Reading recommendations

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1. Characters: start with a calculation

Pavel Etingof et al., *Introduction to representation theory*: start with representations and subrepresentations, then the chapter "Representations of finite groups: basic results" (complete reducibility and characters). *Try:* reconstruct the character table of S_3 from its action on three letters.

2. Geometry: understand the space of lines

Andreas Gathmann, *Algebraic Geometry* (2021/22 notes): begin with affine and projective varieties and the sheaf of regular functions. Use these to learn how local functions glue; leave schemes for later. *Try:* describe the two affine charts of $\mathbb{P}^1$ and count its $\mathbb{F}_q$-points.

3. Topology: learn what cohomology remembers

Allen Hatcher, *Algebraic Topology*: Chapter 0, §1.1 (paths and homotopies), then §2.1 (homology); return to §3.1 for cohomology. This can run alongside step 2. *Try:* compute the homology of $S^2 \simeq \mathbb{P}^1(\mathbb{C})$ and its Euler characteristic.

4. Categories: learn the language through familiar examples

Tom Leinster, *Basic Category Theory*: focus on functors, natural transformations, adjunctions, and limits. Use sets, vector spaces, and groups throughout. *Try:* explain the universal property of a product and the free-group adjunction without diagrams first, then with diagrams.

5. A gentle first encounter with ∞-categories

Start with §§1–2 of Emily Riehl, *Could ∞-category theory be taught to undergraduates?* Then read the introduction and §§2.1, 2.4 of Omar Antolín Camarena, *A Whirlwind Tour of the World of $(\infty,1)$-categories*. These are conceptual entry points; postpone the technical models. *Try:* explain why remembering paths *and their homotopies* gives more information than remembering only homotopy classes of paths.

6. From p-adic groups to their representations

Corinne Blondel, *Basic representation theory of reductive p-adic groups*: after learning the p-adic norm, read §1.1 for matrix groups and compact open subgroups, then §§1.2–1.4 for smooth representations and induction. Take the coefficient field to be $\mathbb{C}$ on a first reading; continue to §2.1 for admissibility. *Try:* put $G = \mathrm{GL}_2(\mathbb{Q}_p)$ and $K_1 = 1 + pM_2(\mathbb{Z}_p)$. Let V be the locally constant $\overline{\mathbb{Q}}_\ell$-valued functions on $\mathbb{P}^1(\mathbb{Q}_p)$, modulo constants ($\ell \neq p$), with $(g \cdot f)(x) = f(g^{-1}x)$. Show that, as a representation of $\mathrm{GL}_2(\mathbb{Z}_p)/K_1 \simeq \mathrm{GL}_2(\mathbb{F}_p)$,

$$V^{K_1} \simeq \mathrm{Fun}\big(\mathbb{P}^1(\mathbb{F}_p), \overline{\mathbb{Q}}_\ell\big) / \{\text{constant functions}\}.$$

Recognize the finite Steinberg representation from the talk. *Hint:* K_1-orbits are the fibers of reduction on projective lines; averaging makes K_1-invariants exact for smooth representations in characteristic zero.

7. Deligne–Lusztig theory through a curve

Chao Li, *Deligne–Lusztig curves*: begin with the sections on representations of $\mathrm{SL}_2(\mathbb{F}_q)$ and the Drinfeld curve $xy^q - x^q y = 1$. Treat étale cohomology as a black box initially; this is an entry to the idea, not a self-contained construction of the theory. *Try:* check that the matrix action of $\mathrm{SL}_2(\mathbb{F}_q)$ preserves the equation.

Back to the talk. Count invariant lines for the four matrix types in $\mathrm{GL}_2(\mathbb{F}_q)$. For a master's reading project, the next bridge is complexes and sheaf cohomology, then étale cohomology and Frobenius. These give the tools to return to the cohomological construction in step 7.