

FALL 2026 - INTRODUCTION TO REPRESENTATION THEORY OF ALGEBRAIC GROUPS IN CHARACTERISTIC p

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0. PLAN

0.1. **Goals.** The goal is to learn about classical results in the representation theory of algebraic groups in characteristic p . Then move on to more modern aspects: understanding Williamson's counterexamples to Lusztig's conjecture, the theory of tilting modules, and the tilting character formula.

0.1.1. *Certain objectives.*

- (i) State and give a proof of Weyl's character formula.
- (ii) State the linkage principle and give an idea of the proof in a special case, as outlined in [Ric26, Chapter 1, §2.5].
- (iii) Prove that $\text{Rep}(G)$ is a highest weight category, and explain the consequences of this fact.
- (iv) Kempf's vanishing theorem and its consequences.

0.1.2. *Tentative objectives.*

- (i) Williamson's construction of counter-examples to Lusztig's character formula conjecture.
- (ii) Understand the theory of tilting modules and their properties.
- (iii) Understand the tilting character formula and its proof.

0.2. **References.** Riche's book in progress [Ric26]. I plan to use this reference as a guide to what we want to understand, but at times it goes too fast, so I will also use Jantzen's book [Jan03] as a reference for more details.

The nice lectures notes [CW20] by Ciappara and Williamson are a streamlined path to Williamson's counterexamples to Lusztig's conjecture.

0.3. **Background.** I will assume some familiarity with algebraic geometry from a first course in the subject. I will also assume some familiarity with algebraic groups, though some material will be quickly reviewed. And some familiarity with a first course in representation theory and some homological algebra would also be helpful.

0.4. **Tentative Schedule.** This is a very rough plan. Specially after the first 6 weeks, we will see where we are.

Lecture	Date	Topic	References
1	Sep. 9	Recap of algebraic groups and characteristic 0 results. Representations of Frobenius Kernel, and Failure of complete reducibility in characteristic p via examples.	[CW20, §1, 2]; [Jan03, §II.1, II.2] [Ric26, Chapter 1, §1]
2	Sep. 16	Induction functors, Weyl modules and costandard modules, and classification of irreducible representations.	[CW20, §2]; [Jan03, §II.2] [Ric26, Chapter 1, §1]
	Sep. 23	<i>No Class – Break</i>	
	Sep. 30	<i>No Class – Break</i>	
	Oct. 7	<i>No Class – Break</i>	
3	Oct. 14	Geometry of flag varieties, Kempf’s vanishing theorem, and consequences. Highest weight category formalism.	[Jan03, §II.4]; [Ric26, Chapter 1, §2] [CW20, §6]
4	Oct. 21	Characters of induced and irreducible representations. Weyl’s character formula: proof and characteristic p considerations.	[Jan03, §II.5]; [Ric26, Chapter 1, §1]
5	Oct. 28	Steinberg’s Tensor Product Theorem. & Introduction to the linkage principle and the affine Weyl group action.	[CW20, §10]; [Jan03, §II.3, II.6] [Ric26, Chapter 1, §2]
6	Nov. 4	Linkage principle (proof of special case)	[Jan03, §II.6] [Ric26, Chapter 1, §2]
7	Nov. 11	Introduction to tilting modules: definitions, properties, and motivation.	
8	Nov. 18	Tilting modules continued and translation functors.	
9	Nov. 25	The tilting character formula and p -Kazhdan-Lusztig polynomials.	
10	Dec. 2	Lusztig’s character formula conjecture: historical context and expectations.	
11	Dec. 9	Williamson’s counter-examples to Lusztig’s conjecture.	
12	Dec. 16	Extra topics: Topics from Oberwolfach program and open questions.	[Jut+23]

REFERENCES

- [CW20] Joshua Ciappara and Geordie Williamson. *Lectures on the Geometry and Modular Representation Theory of Algebraic Groups*. Apr. 30, 2020. arXiv: [2004.14791](#) [[math.RT](#)] ([↑](#) [1](#), [2](#)).
- [Jan03] Jens Carsten Jantzen. *Representations of Algebraic Groups*. 2nd ed. Vol. 107. Mathematical Surveys and Monographs. American Mathematical Society, Providence, RI, 2003, pp. xiv+576 ([↑](#) [1](#), [2](#)).
- [Jut+23] Daniel Juteau, Simon Riche, Wolfgang Soergel, and Geordie Williamson. “Arbeitsgemeinschaft: Geometric Representation Theory”. In: *Oberwolfach Reports* 19.2 (Apr. 13, 2023), pp. 929–978. DOI: [10.4171/owr/2022/18](#). (Visited on 07/28/2026) ([↑](#) [2](#)).
- [Ric26] Simon Riche. “Lectures on Modular Representation Theory of Reductive Algebraic Groups”. Mar. 24, 2026. URL: <https://riche.perso.math.cnrs.fr/curso-riche-total.pdf> ([↑](#) [1](#), [2](#)).